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2.- $f(t) = c$

$\mathcal{L}\{c\} = \int c e^{-st} dt \qquad u = -st \qquad dt = \frac{1}{du} = -\frac{1}{s}$

$du = -s \qquad dv$

$c \cdot \int e^{u} \cdot -\frac{1}{s} du = -\frac{c}{s} \int_0^\infty e^{u} du = -\frac{c}{s} \left[e^{u} \right]_0^\infty = -\frac{c}{s}$

5.- $f(t) = t^3$

$\mathcal{L}\{t^3\} = \int_0^\infty t^3 e^{-st} \qquad u = t^3 \qquad v = -\frac{e^{-st}}{s}$

$du = 3t^2 \qquad dv = e^{-st}$

$= \left[\cancel{t^3 \cdot -\frac{e^{-st}}{s}} \right]_0^\infty - \int_0^\infty -\frac{e^{-st}}{s} \cdot 3t^2 dt$

0

$= \frac{3}{s} \int_0^\infty e^{-st} t^2 dt = \frac{3}{s} \cdot \frac{2}{s^3} = \frac{6}{s^4}$

6.- $f(t) = t^n$

$\mathcal{L}\{t^n\} = \int_0^\infty e^{-st} t^n dt \qquad u = t^n \qquad v = -\frac{e^{-st}}{s}$

$du = nt^{n-1} \qquad dv = e^{-st}$

$= \left[\cancel{\frac{t^n e^{-st}}{s}} \right]_0^\infty - \int_0^\infty -\frac{e^{-st}}{s} \cdot nt^{n-1} dt$

0

$= -\frac{n}{s} \int_0^\infty e^{-st} t^{n-1} dt = \frac{n}{s} \mathcal{L}\{t^{n-1}\} = \frac{n!}{s^{n+1}}$

8.- $f(t) = \cos(t)$

$$\mathcal{L}\{\cos(t)\} = \int_0^{\infty} \cos(t)\, e^{-st}\, dt$$

$$u = \cos(t) \qquad v = -\frac{e^{-st}}{s}$$

$$du = -\operatorname{sen}(t) \qquad dv = e^{-st}$$

$$= -\frac{e^{-st}\cos(t)}{s} - \int \frac{\operatorname{sen}(t)\, e^{-st}}{s}\, dt$$

$$u = -\operatorname{sen}(t) \qquad v = \frac{e^{-st}}{s}$$

$$du = -\cos(t) \qquad dv = e^{-st}$$

$$= -\frac{e^{-st}\cos(t)}{s} - \left(-\frac{e^{-st}\operatorname{sen}(t)}{s} - \int -\frac{e^{-st}\cos(t)}{s^2}\, dt\right)$$

$$= \frac{e^{-st}\operatorname{sen}(t) - s e^{-st}\cos(t)}{s^2+1}\Bigg|_0^{\infty} = \frac{1}{s^2-1}$$

9.- $f(t) = \operatorname{sen}(\alpha t)$

$$\int_0^{\infty} \operatorname{sen}(\alpha t)\, e^{-sx}\, dt = \left[-\frac{1}{s} e^{-sx} \operatorname{sen}(\alpha t)\right]_0^{\infty} + \frac{\alpha}{s}\int_0^{\infty} \cos(\alpha t)\, e^{-st}\, dt$$

$$= \lim_{t \to \infty} \cancel{\left[-\frac{1}{s} e^{-st} \operatorname{sen}(\alpha t)\right]}_{0} + \lim_{t = \infty} \left[\frac{\alpha}{s}\cancel{\left[-\frac{1}{s} e^{-st} \cos(\alpha t)\right]}_{0}\right]$$

$$+ \frac{\alpha}{s^2} - \frac{\alpha^2}{s^2}\int_0^{\infty} \operatorname{sen}(\alpha t)\, e^{-st}\, dt$$

$$\int_0^{\infty} \operatorname{sen}(\alpha t)\, e^{-st}\, dt = \frac{b}{s^2} - \frac{b^2}{s^2}\int_0^{\infty} \operatorname{sen}(\alpha t)\, e^{-st}\, dt$$

$$= \frac{b}{s^2} - \frac{b^2}{s^2}\mathcal{L}\{\operatorname{sen} \alpha t\}$$

$$= \frac{b}{s^2 + b^2}$$

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11.- e^{4t}

$$\mathcal{L}\{e^{4t}\} = \int_0^{\infty} e^{4t} e^{-st} dt = \int_0^{\infty} e^{t(4-s)} dt$$

$u = (4-s)t$
$du = (4-s)t$
$dt = 1/(4-s)t$

$$= \int e^{u} du = \frac{1}{s-4}\left[-e^{-u}\right]_0^{\infty} = \frac{1}{s-4}\left(-e^{-\infty} + e^{0}\right)$$

$$= \frac{1}{s-4}$$

12.- e^{-2t}

$$\mathcal{L}\{e^{-2t}\} = \int_0^{\infty} e^{-2t} e^{-st} dt = \int_0^{\infty} e^{t(-2-s)} dt$$

$u = (-2-s)t$
$du = (-2-s)$
$dt = 1/(-2-s)t$

$$\int e^{u} du = \frac{1}{s+2}\left[-e^{-\theta}\right]_0^{\infty} = \frac{1}{s+2}$$

14 senh(3t)

$$\mathcal{L}\{\operatorname{senh}(3t)\} =$$

$$\int_0^{\infty} e^{-st} \operatorname{senh}(3t)\, dt = \frac{1}{2}\int_0^{\infty} e^{-st}(e^{3t} - e^{-3t})dt$$

$$= \frac{1}{2}\left[\int e^{-st} e^{3t} dt - \int_0^{\infty} e^{-st} e^{-3t} dt = \frac{1}{2}\left[\mathcal{L}\{e^{3t}\} - \mathcal{L}\{e^{-3t}\}\right.\right.$$

$$= \frac{1}{2}\left(\frac{1}{s-3} - \frac{1}{s+3}\right) = \frac{3}{s^2-3^2}$$

15.- $\cosh(6t) =$

$$\cosh(6t) = \frac{1}{2} \cdot e^{6t} + e^{-6t}$$

$$\mathcal{L}\{\cosh(6t)\} = \frac{1}{2}\int_0^{\infty} e^{-st}(e^{6t} + e^{-6t})\,dt$$

$$= \frac{1}{2}\left[\int_0^{\infty} e^{-st} e^{6t}\,dt + \int_0^{\infty} e^{-st} e^{-6t}\,dt\right]$$

$$= \frac{1}{2}\left[\mathcal{L}\{e^{6t}\} + \mathcal{L}\{e^{-6t}\}\right]$$

$$\frac{1}{2}\left[\frac{1}{s-6} + \frac{1}{s+6}\right] = \frac{s}{s^2 - 6^2}$$

17.- $\cosh(\alpha t)$

$$\cosh(\alpha t) = \frac{1}{2} \cdot (e^{\alpha t} + e^{-\alpha t})$$

$$\int_0^{\infty} \cosh(\alpha t) e^{-st}\,dt = \frac{1}{2}\int_0^{\infty} e^{-st}(e^{\alpha t} + e^{-\alpha t})\,dt$$

$$= \frac{1}{2}\left[\int e^{-st} e^{\alpha t}\,dt + \int e^{-st}(e^{-\alpha t})\,dt\right]$$

$$\frac{1}{2}\left[\mathcal{L}\{e^{\alpha t}\} + \mathcal{L}\{e^{-\alpha t}\}\right]$$

$$\frac{1}{2}\left[\frac{1}{s-\alpha} + \frac{1}{s+\alpha}\right] = \frac{s}{s^2 + \alpha^2}$$